The formalism of the stochastic gravity waves scheme: all bugs fixed

Jiandong Liu, Ehouarn Millour

December 15, 2021

1 Formalism and Code Debug

We consider that at each physic step $t \in [t, t + \delta t]$ the vertical velocity w' is represented by a weighted C_n sum of series of (where $n \in N$) monochromatic waves w_n:

$$w^{'} = \sum_{n=1}^{\infty} C_n w_n \tag{1}$$

Where,

$$\sum_{n=1}^{\infty} C_n^2 = 1 \tag{2}$$

$\forall\ n \in N$, the monochromatic wave w_n is as follows:

$$w_n = \Re\{\hat{w}_n(z) e^{z/2H} e^{i(k_n x + l_n y - \omega_n t)}\} \tag{3}$$

$\Re$: the real part of the equation

z: altitude, $z = Hln(P_r/P)$. P pressure and P_r pressure at the reference altitude.

H: atmospheric scale height

i: imaginary index

x: east

y: north

$k_n \in [k_{min} = 2E - 5, k_{max} = 7E - 4]$: east horizontal wave number. $k_n = N/u_r$. Where N is the Brunt-Vaisala frequency and u_r should at the magnitude of the zonal wind at the reference (launch) level. Thus the wavelength of the GW is $\lambda = 2\pi/k_n$.

l_n: north horizontal wave number

ω_n: frequency

t : time

$\hat{w}_n(z)$: vertical wind amplitude at altitude z

To evaluate the wave w_n, we will impose its amplitude $\hat{w}_n$ randomly at a given launching altitude z_0, and then iterate from one model level, z_1 , to the next, z_2 , by a Wentzel-Kramers-Brillouin (WKB) approximation:

$$\hat{w}_n(z_2) = \begin{cases} 0 & \Omega(z_2)\Omega(z_1) < 0 \\ \hat{w}_n(z_1)\sqrt{\frac{m(z_1)}{m(z_2)}}e^{\left(-i\int\limits_{z_1}^{z_2}\left[m(z)-i\frac{\mu m(z)^3}{\rho\Omega}\right]dz\right)} & \hat{w}_n(z_2) < \hat{w}_{n,s} \\ \hat{w}_{n,s} & \hat{w}_n(z_2) \geq \hat{w}_{n,s} \end{cases} \tag{4}$$

$m(z) = N|\vec{k}|/\Omega$ is the vertical wave number [1], with $N^2 = g/T[dT/dz + g/C_p]$ represents the Brunt-Vaisala frequency (code: $N^2 = g/T[dT/dz + g/C_pnew]$ and $N = \sqrt{N^2}$. Notice here the g could be a bug for other planets like Jupiter). $|\vec{k}|$ is the amplitude of the horizontal wave number k_n. $\Omega = \omega - \vec{k}\vec{u}$ is the intrinsic frequency with $\vec{u}$ the zonal wind. μ is the vertical viscosity and $\nu = \mu/\rho$ called kinematic viscosity (dissipation term, tunable). The $\hat{w}_{n,s}$ is the saturation amplitude of a monochromatic wave, which equals to:

$$\hat{w}_{n,s} = \frac{\Omega^2}{|\vec{k}|N}e^{-z/2H}S_c\frac{k^*}{|\vec{k}|} \tag{5}$$

S_c is the saturation parameter that can be given in the program [2]. $k^* = Min(|\vec{k}|, 1/\sqrt{\Delta x \Delta y})$. Δx and Δy is the grid intervals of the model. The term $e^{-z/2H}$ is to take the density decrease effects into account.

By its definition, the Eliassen-Palm (EP) flux (vertical momentum flux of waves) at the lauch level is as follows [3]

$$\vec{F}^z(k, l, \omega) = \Re\left\{\rho_r\frac{\hat{\vec{u}}\hat{w}^*}{2}\right\} = \rho_r\frac{\vec{k}}{2|\vec{k}|^2}m(z)|\hat{w}_n(z)|^2 \tag{6}$$

ρ_r is the mass density at the real altitude. We launch the waves at the top of the convection layers ($p(luanch) = p(ll) if p(ll)/p(1) >$)a tunable constant depends only the specific planet. The EP flux at the reference (launch) level ($\vec{F}^r(k, l, \omega) = \Re\left\{\rho_r\frac{\hat{\vec{u}}\hat{w}^*}{2}\right\} = \rho_r\frac{\vec{k}}{2|\vec{k}|^2}m(z)|\hat{w}_n(z)|^2$) is a tunable value.

Inserting (4) and (5) into (6), we have [4]

$$\vec{F}^{z_2} = \frac{\vec{k}\Omega}{|\vec{k}||\Omega|}\Theta[\Omega(z_2)\Omega(z_1)]Min\left\{|\vec{F}^{z_1}|e^{-2\frac{\mu m^3}{\rho\Omega}\delta z}, \rho_r S_c^2 e^{-\frac{z}{H}}\frac{|\Omega|^3 k^{*2}}{2N|\vec{k}|^4}\right\} \tag{7}$$

Here we have a big bug when we code the second term in Min{} of the equation (7), which in the code it is:

$$S_c^2 e^{-\frac{z}{H}} \frac{|\Omega|^3 k^{*2}}{2N|\vec{k}|^4} \tag{8}$$

The term should be :

$$\rho_r S_c^2 e^{-\frac{z}{H}} \frac{|\Omega|^3 k^{*2}}{2N|\vec{k}|^4} \tag{9}$$

where the ρ_r is the density at the launch altitude(For a special case VENUS or Earth, it is approximate 1. However,other planets are not 1).

Another big bug is trying use pressure to represent ρ in the first term of Min{} of the equation (7). It is coded as:

$$|\vec{F}^{z_1}| e^{-\frac{\mu m^3 p_r}{2p\Omega}\delta z} \tag{10}$$

where the p_r and p is the reference pressure and real pressure at the specific locations, respectively. It works only in Earth and Venus where $\frac{p_r}{p} = \frac{\rho_r T_r}{\rho T}$. The T_r and T are similar and ρ_r approximates 1. However, it will cause sever problems for other planets, in which it makes the critical levels higher than the normal case. Thus real ρ should be used here like equation (7).

Thus we can calculate the EP flux by iteration loop from layer z_1 to z_2, in which $\delta z = z_1 - z_2$ and $z_{ave} = (z_1 + z_2)/2$. Then the tendency $-\frac{1}{\rho}\frac{dF}{dz} = -\frac{1}{\rho}\frac{d\rho\vec{\hat{u}}\hat{w}^*}{dz}$ caused by the waves is as follows:

$$-\frac{1}{\rho}\frac{dF}{dz} = -\frac{1}{\rho}\frac{\delta F}{\delta z} = -\frac{1}{\rho}\frac{\vec{F}^{z_2} - \vec{F}^{z_1}}{z_2 - z_1} \tag{11}$$

Here the code use $g/dp = -1/\rho dz$ to replace. Thus it is a potential bug if the g of the target planets is very big or changes a lot from surface to upper atmosphere. In this case , we should use real density to program here.

Once the tendency is evaluated, we use AR1 to calculate the effects on the zonal wind[5]

$$\left(\frac{\partial \vec{u}}{\partial t}\right)_{GW}^{t+\delta t} = \frac{\delta t}{\Delta t}\frac{1}{M}\sum_{n=1}^{M}\frac{1}{\rho}\frac{dF}{dz} + \frac{\Delta t - \delta t}{\Delta t}\left(\frac{\partial \vec{u}}{\partial t}\right)_{GW}^{t} \tag{12}$$

Thus we have

$$C_n^2 = \left(\frac{\Delta t - \delta t}{\Delta t}\right)^p \frac{\delta t}{M\Delta t} \tag{13}$$

where $p = \lfloor (n-1)/M \rfloor$ is an integer.

2 Other Potential Bugs

1. Every physiq call or physic-time the module starts to "CHARGE" the atmosphere with 8 waves, thus it depends on your physic steps. It takes maybe several sols to complete the "Charging" process that could generate stable effects to the zonal wind and temperature. Therefore,

2. The run start at 0 for a whole year and the run use newstart-generated start (separately) will be not the same anymore.

References

[1] G Gilli, F Forget, A Spiga, T Navarro, E Millour, L Montabone, A Kleinböhl, DM Kass, DJ McCleese, and JT Schofield. Impact of gravity waves on the middle atmosphere of mars: A non-orographic gravity wave parameterization based on global climate modeling and mcs observations. *Journal of Geophysical Research: Planets*, 125(3):e2018JE005873, 2020.

[2] Gabriella Gilli, Sébastien Lebonnois, Francisco González-Galindo, Miguel A López-Valverde, Aurélien Stolzenbach, Franck Lefèvre, Jean-Yves Chaufray, and François Lott. Thermal structure of the upper atmosphere of venus simulated by a ground-to-thermosphere gcm. *Icarus*, 281:55–72, 2017.

[3] Richard S Lindzen. Turbulence and stress owing to gravity wave and tidal breakdown. *Journal of Geophysical Research: Oceans*, 86(C10):9707–9714, 1981.

[4] François Lott and Lionel Guez. A stochastic parameterization of the gravity waves due to convection and its impact on the equatorial stratosphere. *Journal of Geophysical Research: Atmospheres*, 118(16):8897–8909, 2013.

[5] François Lott, Lionel Guez, and P Maury. A stochastic parameterization of non-orographic gravity waves: Formalism and impact on the equatorial stratosphere. *Geophysical Research Letters*, 39(6), 2012.