fftpack5

NAME

FFTPACK5 - a FORTRAN library of fast Fourier transforms

SYNOPSIS

```
Complex Transform Routines
__________________________

CFFT1I    1D complex initialization
CFFT1B    1D complex backward
CFFT1F    1D complex forward

CFFT2I    2D complex initialization
CFFT2B    2D complex backward
CFFT2F    2D complex forward

CFFTMI    multiple complex initialization
CFFTMB    multiple complex backward
CFFTMF    multiple complex forward


Real Transform Routines
_______________________

RFFT1I    1D real initialization
RFFT1B    1D real backward
RFFT1F    1D real forward

RFFT2I    2D real initialization
RFFT2B    2D real backward
RFFT2F    2D real forward
```

RFFTMI   multiple real initialization
RFFTMB   multiple real backward
RFFTMF   multiple real forward

Real Cosine Transform Routines

COST1I   1D real cosine initialization
COST1B   1D real cosine backward
COST1F   1D real cosine forward

COSTMI   multiple real cosine initialization
COSTMB   multiple real cosine backward
COSTMF   multiple real cosine forward

Real Sine Transform Routines

SINT1I   1D real sine initialization
SINT1B   1D real sine backward
SINT1F   1D real sine forward

SINTMI   multiple real sine initialization
SINTMB   multiple real sine backward
SINTMF   multiple real sine forward

Real Quarter-Cosine Transform Routines

COSQ1I   1D real quarter-cosine initialization
COSQ1B   1D real quarter-cosine backward
COSQ1F   1D real quarter-cosine forward

COSQMI   multiple real quarter-cosine initialization
COSQMB   multiple real quarter-cosine backward
COSQMF   multiple real quarter-cosine forward

```
Real Quarter-Sine Transform Routines
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SINQ1I    1D real quarter-sine initialization
SINQ1B    1D real quarter-sine backward
SINQ1F    1D real quarter-sine forward

SINQMI    multiple real quarter-sine initialization
SINQMB    multiple real quarter-sine backward
SINQMF    multiple real quarter-sine forward
```

DESCRIPTION

Library FFTPACK5 contains 1D, 2D, and multiple fast Fourier subroutines, written in Fortran 77, for transforming real and complex data, real even and odd wave data, and real even and odd quarter-wave data. All of the FFTPACK5 routines listed above are grouped in triplets e.g. {CFFT1I, CFFT1F, CFFT1B}. The suffix *I* denotes initialize, *F* denotes forward (as in forward transform) and *B* denotes backward. In an application program, before calling *B* or *F* routines for the first time, or before calling them with a different length, users must initialize an array by calling the *I* routine of the appropriate pair or triplet. Note that *I* routines need not be called each time before a B or F routine is called.

All of the transform routines in FFTPACK5 are normalized.

Error messages are written to unit 6 by routine XERFFT. The standard version of XERFFT issues an error message and halts execution, so that no FFTPACK routine will return to the calling program with error return IER different than zero. Users may consider modifying the STOP statement in order to call system-specific exception-handling facilities.

FFTPACK5 is written in standard Fortran 77 except for several instances where arrays of type REAL or COMPLEX are passed to a subroutine and used as a different type.

References

(1) Vectorizing the Fast Fourier Transforms, by Paul Swarztrauber, Parallel Computations, G. Rodrigue, ed., Academic Press, New York 1982.

(2) Fast Fourier Transforms Algorithms for Vector Computers, by Paul Swarztrauber, Parallel Computing, (1984) pp.45-63.

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cfft1i

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NAME

CFFT1I - initialization routine for CFFT1B and CFFT1F

SYNOPSIS

```
SUBROUTINE CFFT1I (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine CFFT1I initializes array WSAVE for use in
its companion routines CFFT1B and CFFT1F.  Routine CFFT1I must
be called before the first call to  CFFT1B or CFFT1F, and after
whenever the value of integer N changes.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG(REAL(N))) + 4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines CFFT1B or CFFT1F.
```

```
IER      = 0 successful exit
         = 2 input parameter LENSAV not big enough
```

cfft1b

Return to Main Contents

NAME

CFFT1B - complex backward fast Fourier transform

SYNOPSIS

```
 SUBROUTINE CFFT1B (N, INC, C, LENC, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    N, INC, LENC, LENSAV, LENWRK, IER
 COMPLEX    C(LENC)
 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine CFFT1B computes the one-dimensional Fourier
 transform of a single periodic sequence within a complex array.
 This transform is referred to as the backward transform or Fourier
 synthesis, transforming the sequence from spectral to physical
 space.

 This transform is normalized since a call to CFFT1B followed
 by a call to CFFT1F (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 N       Integer length of the sequence to be transformed.  The
         transform is most efficient when N is a product of
         small primes.

 INC     Integer increment between the locations, in array C, of two
         consecutive elements within the sequence to be transformed.
```

C   Complex array of length LENC containing the sequence to be
        transformed.

LENC   Integer dimension of C array. LENC must be at least
        INC*(N-1) + 1.

WSAVE   Real work array with dimension LENSAV. WSAVE's contents
        must be initialized with a call to subroutine CFFT1I before
        the first call to routine CFFT1F or CFFT1B for a given
        transform length N. WSAVE's contents may be re-used for
        subsequent calls to CFFT1F and CFFT1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least
        2*N + INT(LOG(REAL(N))) + 4.

WORK   Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at
        least 2*N.

Output Arguments

C   For index J*INC+1 where J=0,...,N-1,

```
   C(J*INC+1) =

   N-1
   SUM C(K*INC+1)*EXP(I*J*K*2*PI/N)
   K=0
```

        where I=SQRT(-1).

        At other indices, the output value of C does not differ
        from input.

```
IER     =  0 successful exit
        =  1 input parameter LENC   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
```

= 20 input error returned by lower level routine

cfft1f

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NAME

CFFT1F - complex forward fast Fourier transform

SYNOPSIS

```
 SUBROUTINE CFFT1F (N, INC, C, LENC, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    N, INC, LENC, LENSAV, LENWRK, IER
 COMPLEX    C(LENC)
 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine CFFT1F computes the one-dimensional Fourier
 transform of a single periodic sequence within a complex array.
 This transform is referred to as the forward transform or Fourier
 analysis, transforming the sequence from physical to spectral
 space.

 This transform is normalized since a call to CFFT1F followed
 by a call to CFFT1B (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 N       Integer length of the sequence to be transformed.  The
         transform is most efficient when N is a product of
         small primes.

 INC     Integer increment between the locations, in array C, of two
         consecutive elements within the sequence to be transformed.
```

```
C       Complex array of length LENC containing the sequence to be
        transformed.

LENC    Integer dimension of C array.  LENC must be at least
        INC*(N-1) + 1.

WSAVE   Real work array with dimension LENSAV.  WSAVE's contents
        must be initialized with a call to subroutine CFFT1I before
        the first call to routine CFFT1F or CFFT1B for a given
        transform length N.  WSAVE's contents may be re-used for
        subsequent calls to CFFT1F and CFFT1B with the same N.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG(REAL(N))) + 4.


WORK    Real work array of dimension LENWRK.

LENWRK  Integer dimension of WORK array.  LENWRK must be at
        least 2*N.


Output Arguments

C       For index J*INC+1 where J=0,...,N-1 (that is, for the Jth
        element of the sequence),

           C(J*INC+1) =

           N-1
           SUM C(K*INC+1)*EXP(-I*J*K*2*PI/N)
           K=0

        where I=SQRT(-1).

        At other indices, the output value of C does not differ
        from input.

IER     =  0 successful exit
        =  1 input parameter LENC   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        = 20 input error returned by lower level routine
```

cfft2i

Return to Main Contents

NAME

CFFT2I - initialization routine for CFFT2B, CFFT2F

SYNOPSIS

```
SUBROUTINE CFFT2I (L, M, WSAVE, LENSAV, IER)

INTEGER    L, M, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 routine CFFT2I initializes real array WSAVE for use
in its companion routines CFFT2F and CFFT2B for computing two-
dimensional fast Fourier transforms of complex data.  Prime
factorizations of L and M, together with tabulations of the
trigonometric functions, are computed and stored in array WSAVE.

Input Arguments

L       Integer number of elements to be transformed in the first
        dimension.  The transform is most efficient when L is a
        product of small primes.


M       Integer number of elements to be transformed in the second
        dimension.  The transform is most efficient when M is a
        product of small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*(L+M) + INT(LOG(REAL(L))) + INT(LOG(REAL(M))) + 8.
```

Output Arguments

```
WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of L and M, and also containing certain
        trigonometric values which will be used in routines
        CFFT2B or CFFT2F.


WSAVE   Real work array with dimension LENSAV.  The WSAVE array
        must be initialized with a call to subroutine CFFT2I before
        the first call to CFFT2B or CFFT2F, and thereafter whenever
        the values of L, M or the contents of array WSAVE change.
        Using different WSAVE arrays for different transform lengths
        or types in the same program may reduce computation costs
        because the array contents can be re-used.


IER     Integer error return
        =  0 successful exit
        =  2 input parameter LENSAV not big enough
        = 20 input error returned by lower level routine
```

cfft2b

Return to Main Contents

NAME

CFFT2B - complex, two-dimensional backward fast Fourier transform

SYNOPSIS

```
 SUBROUTINE CFFT2B (LDIM, L, M, C, WSAVE, LENSAV,
1                     WORK, LENWRK, IER)

 INTEGER    L, M, LDIM, LENSAV, LENWRK, IER
 COMPLEX    C(LDIM,M)
 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine CFFT2B computes the two-dimensional discrete
 Fourier transform of a complex periodic array.  This transform is
 known as the backward transform or Fourier synthesis, transforming
 from spectral to physical space.

 Routine CFFT2B is normalized, in that a call to CFFT2B followed
 by a call to CFFT2F (or vice-versa) reproduces the original array
 within roundoff error.

 Input Arguments

 LDIM    Integer first dimension of two-dimensional complex array C.


 L       Integer number of elements to be transformed in the first
         dimension of the two-dimensional complex array C.  The value
         of L must be less than or equal to that of LDIM.  The
         transform is most efficient when L is a product of small
         primes.
```

```
M       Integer number of elements to be transformed in the second
        dimension of the two-dimensional complex array C.  The
        transform is most efficient when M is a product of small
        primes.

C       Complex array of two dimensions containing the (L,M) subarray
        to be transformed.  C's first dimension is LDIM, its second
        dimension must be at least M.


WSAVE   Real work array with dimension LENSAV.  WSAVE's contents
        must be initialized with a call to subroutine CFFT2I before
        the first call to routine CFFT2F or CFFT2B with transform
        lengths L and M.  WSAVE's contents may be re-used for
        subsequent calls to CFFT2F and CFFT2B with the same
        transform lengths L and M.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*(L+M) + INT(LOG(REAL(L))) + INT(LOG(REAL(M))) + 8.

WORK    Real work array.

LENWRK  Integer dimension of WORK array.  LENWRK must be at least
        2*L*M.

Output Arguments


 C      Complex output array.  For purposes of exposition,
        assume the index ranges of array C are defined by
        C(0:L-1,0:M-1).

        For I=0,...,L-1 and J=0,...,M-1, the C(I,J)'s are given
         in the traditional aliased form by

                  L-1  M-1
         C(I,J) = SUM  SUM  C(L1,M1)*
                  L1=0 M1=0

                    EXP(SQRT(-1)*2*PI*(I*L1/L + J*M1/M))
```

And in unaliased form, the C(I,J)'s are given by

```
         LF    MF
C(I,J) = SUM   SUM   C(L1,M1,K1)*
         L1=LS M1=MS

           EXP(SQRT(-1)*2*PI*(I*L1/L + J*M1/M))
```

where

```
   LS= -L/2    and LF=L/2-1   if L is even;
   LS=-(L-1)/2 and LF=(L-1)/2 if L is odd;
   MS= -M/2    and MF=M/2-1   if M is even;
   MS=-(M-1)/2 and MF=(M-1)/2 if M is odd;
```

and

```
   C(L1,M1) = C(L1+L,M1) if L1 is zero or negative;
   C(L1,M1) = C(L1,M1+M) if M1 is zero or negative;
```

The two forms give different results when used to interpolate between elements of the sequence.

```
IER     Integer error return
        =  0 successful exit
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        =  5 input parameter L > LDIM
        = 20 input error returned by lower level routine
```

cfft2f

Return to Main Contents

NAME

CFFT2F - complex, two-dimensional forward fast Fourier transform

SYNOPSIS

```
 SUBROUTINE CFFT2F (LDIM, L, M, C, WSAVE, LENSAV,
1                     WORK, LENWRK, IER)

 INTEGER    L, M, LDIM, LENSAV, LENWRK, IER
 COMPLEX    C(LDIM,M)
 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine CFFT2F computes the two-dimensional discrete
 Fourier transform of a complex periodic array.  This transform is
 known as the forward transform or Fourier analysis, transforming
 from physical to spectral space.

 Routine CFFT2F is normalized, in that a call to CFFT2F followed
 by a call to CFFT2B (or vice-versa) reproduces the original array
 within roundoff error.


 Input Arguments


 LDIM    Integer first dimension of two-dimensional complex array C.


 L       Integer number of elements to be transformed in the first
```

dimension of the two-dimensional complex array C. The value of L must be less than or equal to that of LDIM. The transform is most efficient when L is a product of small primes.

M — Integer number of elements to be transformed in the second dimension of the two-dimensional complex array C. The transform is most efficient when M is a product of small primes.

C — Complex array of two dimensions containing the (L,M) subarray to be transformed. C's first dimension is LDIM, its second dimension must be at least M.

WSAVE — Real work array with dimension LENSAV. WSAVE's contents must be initialized with a call to subroutine CFFT2I before the first call to routine CFFT2F or CFFT2B with transform lengths L and M. WSAVE's contents may be re-used for subsequent calls to CFFT2F and CFFT2B having those same transform lengths.

LENSAV — Integer dimension of WSAVE array. LENSAV must be at least 2*(L+M) + INT(LOG(REAL(L))) + INT(LOG(REAL(M))) + 8.

WORK — Real work array.

LENWRK — Integer dimension of WORK array. LENWRK must be at least 2*L*M.

Output Arguments

C — Complex output array. For purposes of exposition, assume the index ranges of array C are defined by C(0:L-1,0:M-1).

```
         For I=0,...,L-1 and J=0,...,M-1, the C(I,J)'s are given
         in the traditional aliased form by

                            L-1  M-1
         C(I,J) =   1/(L*M)*SUM  SUM  C(L1,M1)*
                            L1=0 M1=0

                    EXP(-SQRT(-1)*2*PI*(I*L1/L + J*M1/M))

         And in unaliased form, the C(I,J)'s are given by

                           LF    MF
         C(I,J) =  1/(L*M)*SUM   SUM   C(L1,M1)*
                           L1=LS M1=MS

                    EXP(-SQRT(-1)*2*PI*(I*L1/L + J*M1/M))

         where

            LS= -L/2    and LF=L/2-1   if L is even;
            LS=-(L-1)/2 and LF=(L-1)/2 if L is odd;
            MS= -M/2    and MF=M/2-1   if M is even;
            MS=-(M-1)/2 and MF=(M-1)/2 if M is odd;

         and

            C(L1,M1) = C(L1+L,M1) if L1 is zero or negative;
            C(L1,M1) = C(L1,M1+M) if M1 is zero or negative;

         The two forms give different results when used to
         interpolate between elements of the sequence.

 IER     Integer error return
         =  0 successful exit
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
         =  5 input parameter L > LDIM
         = 20 input error returned by lower level routine
```

cfftmi

Return to Main Contents

NAME

CFFTMI - initialization routine for CFFTMB and CFFTMF

SYNOPSIS

```
SUBROUTINE CFFTMI (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine CFFTMI initializes array WSAVE for use in
its companion routines CFFTMB and CFFTMF.  Routine CFFTMI must
be called before the first call to  CFFTMB or CFFTMF, and after
whenever the value of integer N changes.

Input Arguments

N        Integer length of each sequence to be transformed.  The
         transform is most efficient when N is a product of
         small primes.

LENSAV   Integer dimension of WSAVE array.  LENSAV must be at least
         2*N + INT(LOG(REAL(N))) + 4.


Output Arguments

WSAVE    Real work array with dimension LENSAV, containing the
         prime factors of N and also containing certain trigonometric
         values which will be used in routines CFFTMB or CFFTMF.
```

```
IER     = 0 successful exit
        = 2 input parameter LENSAV not big enough
```

cfftmb

Return to Main Contents

NAME

CFFTMB - complex, multiple backward fast Fourier transform

SYNOPSIS

```
 SUBROUTINE CFFTMB (LOT, JUMP, N, INC, C, LENC, WSAVE, LENSAV,
1                  WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENC, LENSAV, LENWRK, IER
 COMPLEX    C(LENC)
 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine CFFTMB computes the one-dimensional Fourier
 transform of multiple periodic sequences within a complex array.
 This transform is referred to as the backward transform or Fourier
 synthesis, transforming the sequences from spectral to physical
 space.

 This transform is normalized since a call to CFFTMF followed
 by a call to CFFTMB (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array C.

 JUMP    Integer increment between the locations, in array C,
         of the first elements of two consecutive sequences
         to be transformed.
```

N   Integer length of each sequence to be transformed. The transform is most efficient when N is a product of small primes.

INC   Integer increment between the locations, in array C, of two consecutive elements within the same sequence to be transformed.

C   Complex array containing LOT sequences, each having length N, to be transformed. C can have any number of dimensions, but the total number of locations must be at least LENC.

LENC   Integer dimension of C array. LENC must be at least (LOT-1)*JUMP + INC*(N-1) + 1.

WSAVE   Real work array of length LENSAV. WSAVE's contents must be initialized with a call to subroutine CFFTMI before the first call to routine CFFTMF or CFFTMB for a given transform length N.

LENSAV   Integer dimension of WSAVE array. LENSAV must be at least 2*N + INT(LOG(REAL(N))) + 4.

WORK   Real work array of dimension LENWRK.

LENWRK   Integer dimension of WORK array. LENWRK must be at least 2*LOT*N.

Output Arguments

C   For index L*JUMP+J*INC+1 where J=0,...,N-1 and L=0,...,LOT-1, (that is, for the Jth element of the Lth sequence),

```
   C(L*JUMP+J*INC+1) =

   N-1
   SUM C(L*JUMP+K*INC+1)*EXP(I*J*K*2*PI/N)
```

```
            K=0

         where I=SQRT(-1).

         At other indices, the output value of C does not differ
         from input.

IER      = 0 successful exit
         = 1 input parameter LENC   not big enough
         = 2 input parameter LENSAV not big enough
         = 3 input parameter LENWRK not big enough
         = 4 input parameters INC,JUMP,N,LOT are not consistent.

             The parameters integers INC, JUMP, N and LOT are
             consistent if equality
             I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
             and J1,J2 < LOT implies I1=I2 and J1=J2.

             For multiple FFTs to execute correctly, input variables
             INC, JUMP, N and LOT must be consistent ... otherwise at
             least one array element mistakenly is transformed more
             than once.
```

cfftmf

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NAME

CFFTMF - complex, multiple forward fast Fourier transform

SYNOPSIS

```
 SUBROUTINE CFFTMF (LOT, JUMP, N, INC, C, LENC, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENC, LENSAV, LENWRK, IER
 COMPLEX    C(LENC)
 REAL       WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine CFFTMF computes the one-dimensional Fourier
 transform of multiple periodic sequences within a complex array.
 This transform is referred to as the forward transform or Fourier
 analysis, transforming the sequences from physical to spectral
 space.

 This transform is normalized since a call to CFFTMF followed
 by a call to CFFTMB (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array C.

 JUMP    Integer increment between the locations, in array C,
         of the first elements of two consecutive sequences
         to be transformed.
```

N Integer length of each sequence to be transformed. The transform is most efficient when N is a product of small primes.

INC Integer increment between the locations, in array C, of two consecutive elements within the same sequence to be transformed.

C Complex array containing LOT sequences, each having length N, to be transformed. C can have any number of dimensions, but the total number of locations must be at least LENC.

LENC Integer dimension of C array. LENC must be at least (LOT-1)*JUMP + INC*(N-1) + 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must be initialized with a call to subroutine CFFTMI before the first call to routine CFFTMF or CFFTMB for a given transform length N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least 2*N + INT(LOG(REAL(N))) + 4.

WORK Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least 2*LOT*N.

Output Arguments

C For index L*JUMP + J*INC +1 where J=0,...,N-1 and L=0,...,LOT-1, (that is, for the Jth element of the Lth sequence),

```
C(L*JUMP+J*INC+1) =

N-1
SUM C(L*JUMP+K*INC+1)*EXP(-I*J*K*2*PI/N)
```

```
            K=0

         where I=SQRT(-1).

         At other indices, the output value of C does not differ
         from input.

 IER     = 0 successful exit
         = 1 input parameter LENC   not big enough
         = 2 input parameter LENSAV not big enough
         = 3 input parameter LENWRK not big enough
         = 4 input parameters INC,JUMP,N,LOT are not consistent.

             The parameters integers INC, JUMP, N and LOT are
             consistent if equality
             I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
             and J1,J2 < LOT implies I1=I2 and J1=J2.

             For multiple FFTs to execute correctly, input variables
             INC, JUMP, N and LOT must be consistent ... otherwise at
             least one array element mistakenly is transformed more
             than once.
```

rfft1i

Return to Main Contents

NAME

RFFT1I - initialization routine for RFFT1B and RFFT1F

SYNOPSIS

```
SUBROUTINE RFFT1I (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine RFFT1I initializes array WSAVE for use
in its companion routines RFFT1B and RFFT1F.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        N + INT(LOG (REAL(N))) +4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines RFFT1B or RFFT1F.
```

```
IER      =  0 successful exit
         =  2 input parameter LENSAV not big enough
```

rfft1b

Return to Main Contents

NAME

RFFT1B - real backward fast Fourier transform

SYNOPSIS

```
SUBROUTINE RFFT1B (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine RFFT1B computes the one-dimensional Fourier
transform of a periodic sequence within a real array.  This
is referred to as the backward transform or Fourier synthesis,
transforming the sequence from spectral to physical space.


This transform is normalized since a call to RFFT1B followed
by a call to RFFT1F (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
        transformed.
```

LENR   Integer dimension of R array. LENR must be at least INC*(N-1) + 1.

WSAVE   Real work array o length LENSAV. WSAVE's contents must be initialized with a call to subroutine RFFT1I before the first call to routine RFFT1F or RFFT1B for a given transform length N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least N + INT(LOG (REAL(N))) +4.

WORK   Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at N.

Output Arguments

R   Real output array R. For purposes of exposition, assume R's range of indices is given by R(0:(N-1)*INC).

The output values of R are written over the input values. If N is even, set NH=N/2-1; then for J=0,...,N-1

```
 R(J*INC) =     R(0) +

    [(-1)**J*R((N-1)*INC)]

               NH
      +        SUM  R((2*N1-1)*INC)*COS(J*N1*2*PI/N)
               N1=1

               NH
      +        SUM  R(2*N1*INC)*SIN(J*N1*2*PI/N)
               N1=1
```

If N is odd, set NH=(N-1)/2 and define R as above, except remove the expression in square brackets [].

```
IER      Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big enough
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
```

rfft1f

Return to Main Contents

NAME

RFFT1F - real backward fast Fourier transform

SYNOPSIS

```
SUBROUTINE RFFT1F (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine RFFT1F computes the one-dimensional Fourier
transform of a periodic sequence within a real array.  This
is referred to as the forward transform or Fourier analysis,
transforming the sequence from physical to spectral space.


This transform is normalized since a call to RFFT1F followed
by a call to RFFT1B (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
        transformed.
```

LENR   Integer dimension of R array. LENR must be at least INC*(N-1) + 1.

WSAVE   Real work array of length LENSAV. WSAVE's contents must be initialized with a call to subroutine RFFT1I before the first call to routine RFFT1F or RFFT1B for a given transform length N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least N + INT(LOG (REAL(N))) +4.

WORK   Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at N.

Output Arguments

R   Real output array R. For purposes of exposition, assume R's range of indices is given by R(0:(N-1)*INC).

Then

```
                       N-1
        R(0) =         SUM  R(N1*INC)/N
                       N1=0
```

If N is even, set NH=N/2-1; if N is odd set NH=(N-1)/2; then for J=1,...,NH

```
          R((2*J-1)*INC) =

                    N-1
                 2.*SUM  (R(N1*INC)*COS(J*N1*2*PI/N)/N
                   N1=0
```

and

```
          R(2*J*INC) =

                   N-1
                2.*SUM  (R(N1*INC)*SIN(J*N1*2*PI/N)/N
                  N1=0

        Also if N is even then

          R((N-1)*INC) =

                   N-1
                   SUM  (-1)**N1*R(N1*INC)/N
                   N1=0


IER      Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big enough
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
```

rfft2i

Return to Main Contents

NAME

RFFT2I - initialization routine for RFFT2B and RFFT2F

SYNOPSIS

```
SUBROUTINE RFFT2I (L, M, WSAVE, LENSAV, IER)
INTEGER    L, M, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 routine RFFT2I initializes real array WSAVE for use
in its companion routines RFFT2F and RFFT2B for computing the two-
dimensional fast Fourier transform of real data.  Prime
factorizations of L and M, together with tabulations of the
trigonometric functions, are computed and stored in array WSAVE.
RFFT2I must be called prior to the first call to RFFT2F or RFFT2B.
Separate WSAVE arrays are required for different values of L or M.

Input Arguments

L       Integer number of elements to be transformed in the first
        dimension.  The transform is most efficient when L is a
        product of small primes.

M       Integer number of elements to be transformed in the second
        dimension.  The transform is most efficient when M is a
        product of small primes.

LENSAV  Integer number of elements in the WSAVE array.  LENSAV must
        be at least L + M + INT(LOG(REAL(L))) + INT(LOG(REAL(M))) +8.
```

Output Arguments

```
WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of L and M, and also containing certain
        trigonometric values which will be used in routines
        RFFT2B or RFFT2F.


IER     Integer error return
        =  0 successful exit
        =  2 input parameter LENSAV not big enough
        = 20 input error returned by lower level routine
```

rfft2b

Return to Main Contents

NAME

RFFT2B - complex to real, two-dimensional backward fast Fourier transform

SYNOPSIS

```
SUBROUTINE RFFT2B (LDIM, L, M, R, WSAVE, LENSAV, WORK, LENWRK, IER)
INTEGER    LDIM, L, M, LENSAV, LENWRK, IER
REAL       R(LDIM,M), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine RFFT2B computes the two-dimensional discrete
Fourier transform of the complex Fourier coefficients a real
periodic array.  This transform is known as the backward transform
or Fourier synthesis, transforming from spectral to physical space.

Routine RFFT2B is normalized: a call to RFFT2B followed by a
call to RFFT2F (or vice-versa) reproduces the original array
within roundoff error.

Input Arguments

LDIM    Integer first dimension of the two-dimensional real
        array R, which must be at least 2*(L/2+1).

L       Integer number of elements to be transformed in the first
        dimension of the two-dimensional real array R.  The value
        of L must be less than or equal to that of LDIM.  The
        transform is most efficient when L is a product of small
        primes.

M       Integer number of elements to be transformed in the second
```

dimension of the two-dimensional real array R. The transform is most efficient when M is a product of small primes.

R — Real array of two dimensions containing the L/2+1-by-M complex subarray of spectral coefficients. R's first dimension is LDIM and its second dimension must be at least as large as M.

WSAVE — Real work array of length LENSAV. WSAVE's contents must be initialized with a call to subroutine RFFT2I before the first call to routine RFFT2F or RFFT2B with lengths L and M. WSAVE's contents may be re-used for subsequent calls to RFFT2F and RFFT2B with the same transform lengths L and M.

LENSAV — Integer number of elements in the WSAVE array. LENSAV must be at least L + M + INT(LOG(REAL(L))) + INT(LOG(REAL(M))) +8.

WORK — Real array of dimension LENWRK, where LENWRK is defined below. WORK provides workspace, and its contents need not be saved between calls to routines RFFT2B and RFFT2F.

LENWRK — Integer number of elements in the WORK array. LENWRK must be at least LDIM*M.

Output Arguments

R — Real output array R of size LDIM-by-M, where LDIM is at least L. For purposes of exposition, assume the index ranges of array R are defined by R(0:L-1,0:M-1), and the complex Fouier coefficient array by C(0:L/2,0:M-2).

```
          L/2  M-1
 R(I,J) = SUM  SUM  C(L1,M1)
          L1=0 M1=0

     *EXP(SQRT(-1)*2*PI*(I*L1/L+J*M1/M))

   L-1  M-1
```

```
        + SUM  SUM  CONJ(C(L1,M1))
     L1=L/2+1  M1=0

             *EXP(SQRT(-1)*2*PI*(I*(L-L1)/L+J*M1/M))


IER      Integer error return
         =  0 successful exit
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
         =  6 input parameter LDIM < 2*(L/2+1)
         = 20 input error returned by lower level routine
```

rfft2f

[Return to Main Contents]

NAME

RFFT2F - real to complex, two-dimensional forward fast Fourier transform

SYNOPSIS

```
SUBROUTINE RFFT2F (LDIM, L, M, R, WSAVE, LENSAV, WORK, LENWRK, IER)
INTEGER    LDIM, L, M,  LENSAV, LENWRK, IER
REAL       R(LDIM,M), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine RFFT2F computes the two-dimensional discrete
Fourier transform of a real periodic array.  This transform is
known as the forward transform or Fourier analysis, transforming
from physical to spectral space.

Routine RFFT2F is normalized: a call to RFFT2F followed by a
call to RFFT2B (or vice-versa) reproduces the original array
within roundoff error.

Input Arguments

LDIM    Integer first dimension of the two-dimensional real
        array R, which must be at least 2*(L/2+1).

L       Integer number of elements to be transformed in the first
        dimension of the two-dimensional real array R.  The value
        of L must be less than or equal to that of LDIM.  The
        transform is most efficient when L is a product of small
        primes.

M       Integer number of elements to be transformed in the second
```

dimension of the two-dimensional real array R. The transform is most efficient when M is a product of small primes.

R Real array of two dimensions containing the L-by-M subarray to be transformed. R's first dimension is LDIM and its second dimension must be at least as large as M.

WSAVE Real work array of length LENSAV. WSAVE's contents must be initialized with a call to subroutine RFFT2I before the first call to routine RFFT2F or RFFT2B with lengths L and M. WSAVE's contents may be re-used for subsequent calls to RFFT2F and RFFT2B with the same transform lengths.

LENSAV Integer number of elements in the WSAVE array. LENSAV must be at least L + M + INT(LOG(REAL(L))) + INT(LOG(REAL(M))) +8.

WORK Real array of dimension LENWRK which is defined below. WORK provides workspace, and its contents need not be saved between calls to routines RFFT2F and RFFT2B.

LENWRK Integer number of elements in the WORK array. LENWRK must be at least LDIM*M.

Output Arguments

R Real output array of two dimensions. Only half of the Fourier spectrum of R is computed and stored as a L/2+1-by- M complex array. The L wavenumbers stored are 0 through L/2+1. The leading dimension of R LDIM must be at least 2*(L/2+1).

For purposes of exposition, assume the index ranges of a complex array C are defined by C(0:L/2,0:M-1).

For I=0,...,L/2 and J=0,...,M-1, the C(I,J)'s are given in the traditional aliased form by

L-1 M-1

```
        C(I,J) =   1/(L*M)*SUM  SUM  C(L1,M1)*
                           L1=0 M1=0

                   EXP(-SQRT(-1)*2*PI*(I*L1/L + J*M1/M))

        The complex C(I,J), I=0,...,L/2 and J=0,...,M-1 are stored
        in the real array R as:

            Re(C(I,J)) = R(2*I+1,J+1)
            Im(C(I,J)) = R(2*I+2,J+1).

IER     Integer error return
        =  0 successful exit
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        =  6 input parameter LDIM < 2*(L+1)
        = 20 input error returned by lower level routine
```

rfftmi

Return to Main Contents

NAME

RFFTMI - initialization routine for RFFTMB and RFFTMF

SYNOPSIS

```
SUBROUTINE RFFTMI (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine RFFTMI initializes array WSAVE for use
in its companion routines RFFTMB and RFFTMF.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N       Integer length of each sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        N + INT(LOG (REAL(N))) +4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines RFFTMB or RFFTMF.
```

```
IER      =  0 successful exit
         =  2 input parameter LENSAV not big enough
```

rfftmb

Return to Main Contents

NAME

RFFTMB - real, multiple backward fast Fourier transform

SYNOPSIS

```
 SUBROUTINE RFFTMB (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV)     ,WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine RFFTMB computes the one-dimensional Fourier
 transform of multiple periodic sequences within a real array.
 This transform is referred to as the backward transform or Fourier
 synthesis, transforming the sequences from spectral to physical
 space.

 This transform is normalized since a call to RFFTMB followed
 by a call to RFFTMF (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences
         to be transformed.

 N       Integer length of each sequence to be transformed.  The
```

transform is most efficient when N is a product of
small primes.

INC   Integer increment between the locations, in array R, of two consecutive elements within the same sequence.

R   Real array containing LOT sequences, each having length N. R can have any number of dimensions, but the total number of locations must be at least LENR.

LENR   Integer dimension of R array. LENR must be at least (LOT-1)*JUMP + INC*(N-1) + 1.

WSAVE   Real work array of length LENSAV. WSAVE's contents must be initialized with a call to subroutine RFFTMI before the first call to routine RFFTMF or RFFTMB for a given transform length N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least N + INT(LOG (REAL(N))) +4.

WORK   Real work array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least LOT*N.

Output Arguments

R   Real output array R. For purposes of exposition, assume R's range of indices is given by R(0:(LOT-1)*JUMP+(N-1)*INC).

The output values of R are written over the input values. If N is even, set NH=N/2-1; then for I=0,...,LOT-1 and J=0,...,N-1

```
 R(I*JUMP+J*INC) =     R(I*JUMP) +

    [(-1)**J*R(I*JUMP+(N-1)*INC)]
```

```
                  NH
            +     SUM  R(I*JUMP+(2*N1-1)*INC)*COS(J*N1*2*PI/N)
                  N1=1

                  NH
            +     SUM  R(I*JUMP+2*N1*INC)*SIN(J*N1*2*PI/N)
                  N1=1

         If N is odd, set NH=(N-1)/2 and define R as above,
         except remove the expression in square brackets [].

IER     Integer error return
        =  0 successful exit
        =  1 input parameter LENR   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        =  4 input parameters INC,JUMP,N,LOT are not consistent.

             The parameters integers INC, JUMP, N and LOT are
             consistent if equality
             I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
             and J1,J2 < LOT implies I1=I2 and J1=J2.

             For multiple FFTs to execute correctly, input variables
             INC, JUMP, N and LOT must be consistent ... otherwise at
             least one array element mistakenly is transformed more
             than once.
```

rfftmf

Return to Main Contents

NAME

RFFTMF - real, multiple forward fast Fourier transform

SYNOPSIS

```
 SUBROUTINE RFFTMF (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV)     ,WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine RFFTMF computes the one-dimensional Fourier
 transform of multiple periodic sequences within a real array.
 This transform is referred to as the forward transform or Fourier
 analysis, transforming the sequences from physical to spectral
 space.

 This transform is normalized since a call to RFFTMF followed
 by a call to RFFTMB (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences
         to be transformed.

 N       Integer length of each sequence to be transformed.  The
```

```
         transform is most efficient when N is a product of
         small primes.

INC      Integer increment between the locations, in array R,
         of two consecutive elements within the same sequence.

R        Real array containing LOT sequences, each having length N.
         R can have any number of dimensions, but the total number
         of locations must be at least LENR.

LENR     Integer dimension of R array.  LENR must be at
         least (LOT-1)*JUMP + INC*(N-1) + 1.


WSAVE    Real work array o length LENSAV.  WSAVE's contents must
         be initialized with a call to subroutine RFFTMI before the
         first call to routine RFFTMF or RFFTMB for a given transform
         length N.

LENSAV   Integer dimension of WSAVE array.  LENSAV must be at least
         N + INT(LOG (REAL(N))) +4.


WORK     Real work array of dimension LENWRK.

LENWRK   Integer dimension of WORK array.  LENWRK must be at
         least LOT*N.


Output Arguments

 R       Real output array R.  For purposes of exposition,
         assume R's range of indices is given by
         R(0:(LOT-1)*JUMP+(N-1)*INC).

         Then for I=0,...,LOT-1

                        N-1
         R(I*JUMP) =    SUM  R(I*JUMP+N1*INC)/N
                        N1=0

         If N is even, set NH=N/2-1; if N is odd set NH=(N-1)/2;
         then for J=1,...,NH
```

```
          R(I*JUMP+(2*J-1)*INC) =

                    N-1
                 2.*SUM  (R(I*JUMP+N1*INC)*COS(J*N1*2*PI/N)/N
                   N1=0

         and

           R(I*JUMP+2*J*INC) =

                    N-1
                 2.*SUM  (R(I*JUMP+N1*INC)*SIN(J*N1*2*PI/N)/N
                   N1=0

         Also if N is even then

           R(I*JUMP+(N-1)*INC) =

                    N-1
                    SUM  (-1)**N1*R(I*JUMP+N1*INC)/N
                    N1=0

IER     Integer error return
        =  0 successful exit
        =  1 input parameter LENR   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        =  4 input parameters INC,JUMP,N,LOT are not consistent.

             The parameters integers INC, JUMP, N and LOT are
             consistent if equality
             I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
             and J1,J2 < LOT implies I1=I2 and J1=J2.

             For multiple FFTs to execute correctly, input variables
             INC, JUMP, N and LOT must be consistent ... otherwise at
             least one array element mistakenly is transformed more
             than once.
```

cost1i

Return to Main Contents

NAME

COST1I - initialization routine for COST1B and COST1F

SYNOPSIS

```
SUBROUTINE COST1I (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine COST1I initializes array WSAVE for use
in its companion routines COST1F and COST1B.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N-1 is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG (REAL(N))) +4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines COST1B or COST1F.
```

```
IER      Integer error return
         =  0 successful exit
         =  2 input parameter LENSAV not big enough
         = 20 input error returned by lower level routine
```

cost1b

Return to Main Contents

NAME

COST1B - real backward cosine fast Fourier transform

SYNOPSIS

```
SUBROUTINE COST1B (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine COST1B computes the one-dimensional Fourier
transform of an even sequence within a real array.  This
transform is referred to as the backward transform or Fourier
synthesis, transforming the sequence from spectral to physical
space.

This transform is normalized since a call to COST1B followed
by a call to COST1F (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N-1 is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
        transformed.
```

```
LENR     Integer dimension of R array.  LENR must be at least
         INC*(N-1)+ 1.


WSAVE    Real work array of length LENSAV.  WSAVE's contents must
         be initialized with a call to subroutine COST1I before the
         first call to routine COST1F or COST1B for a given transform
         length N.  WSAVE's contents may be re-used for subsequent
         calls to COST1F and COST1B with the same N.


LENSAV   Integer dimension of WSAVE array.  LENSAV must be at least
         2*N + INT(LOG (REAL(N))) +4.


WORK     Real work array of dimension at least LENWRK.

LENWRK   Integer dimension of WORK array.  LENWRK must be at
         least N-1.


Output Arguments

R        Real output array R.  For purposes of exposition,
         assume R's range of indices is given by
         R(0:(N-1)*INC).

         The output values of R are written over the input values.
         For J=0,...,N-1

          R(J*INC) =

              N-1
              SUM  R(N1*INC)*COS(J*N1*PI/(N-1))
              N1=0

IER      Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big enough
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
         = 20 input error returned by lower level routine
```

cost1f

Return to Main Contents

NAME

COST1F - real backward cosine fast Fourier transform

SYNOPSIS

```
SUBROUTINE COST1F (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine COST1F computes the one-dimensional Fourier
transform of an even sequence within a real array.  This
transform is referred to as the forward transform or Fourier
analysis, transforming the sequence from  physical to spectral
space.

This transform is normalized since a call to COST1F followed
by a call to COST1B (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N-1 is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
        transformed.
```

LENR   Integer dimension of R array. LENR must be at least INC*(N-1)+ 1.

WSAVE   Real work array of length LENSAV. WSAVE's contents must be initialized with a call to subroutine COST1I before the first call to routine COST1F or COST1B for a given transform length N. WSAVE's contents may be re-used for subsequent calls to COST1F and COST1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least 2*N + INT(LOG (REAL(N))) +4.

WORK   Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least N-1.

Output Arguments

R   Real output array R. For purposes of exposition, assume R's range of indices is given by R(0:(N-1)*INC).

The output values of R are written over the input values.

```
 R(0) =

     0.5*X(0)/(N-1)

     N-2
   + SUM  R(N1*INC)/(N-1)
     N1=1

   + 0.5*X((N-1)*INC)/(N-1)

For J=1,...,N-2
 R(J*INC) =
```

```
           R(0)/(N-1)

           N-2
         + SUM  2.0*(X(N1*INC)*COS(J*N1*PI/(N-1)))/(N-1)
           N1=1

         + ((-1)**J)*X((N-1)*INC)/(N-1)

       R((N-1)*INC) =

           0.5*X(0)/(N-1)

           N-2
         + SUM  R(N1*INC)*((-1)**N1)/(N-1)
           N1=1

         + 0.5*((-1)**(N-1))*X((N-1)*INC)/(N-1)

IER     Integer error return
        =  0 successful exit
        =  1 input parameter LENR   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        = 20 input error returned by lower level routine
```

costmi

Return to Main Contents

NAME

COSTMI - initialization routine for COSTMB and COSTMF

SYNOPSIS

```
SUBROUTINE COSTMI (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine COSTMI initializes array WSAVE for use
in its companion routines COSTMF and COSTMB.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N        Integer length of each sequence to be transformed.  The
         transform is most efficient when N is a product of
         small primes.

LENSAV   Integer dimension of WSAVE array.  LENSAV must be at least
         2*N + INT(LOG (REAL(N))) +4


Output Arguments

WSAVE    Real work array with dimension LENSAV, containing the
         prime factors of N and also containing certain trigonometric
         values which will be used in routines COSTMB or COSTMF.
```

```
IER       Integer error return
          =  0 successful exit
          =  2 input parameter LENSAV not big enough
          = 20 input error returned by lower level routine
```

costmb

Return to Main Contents

NAME

COSTMB - real, multiple backward cosine fast Fourier transform

SYNOPSIS

```
 SUBROUTINE COSTMB (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine COSTMB computes the one-dimensional Fourier
 transform of multiple even sequences within a real array.  This
 transform is referred to as the backward transform or Fourier
 synthesis, transforming the sequences from spectral to physical
 space.

 This transform is normalized since a call to COSTMB followed
 by a call to COSTMF (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences
         to be transformed.

 N       Integer length of each sequence to be transformed.  The
```

transform is most efficient when N-1 is a product of
small primes.

```
INC     Integer increment between the locations, in array R,
        of two consecutive elements within the same sequence.

R       Real array containing LOT sequences, each having length N.
        R can have any number of dimensions, but the total number
        of locations must be at least LENR.

LENR    Integer dimension of R array.  LENR must be at least
        (LOT-1)*JUMP + INC*(N-1)+ 1.


WSAVE   Real work array of length LENSAV.  WSAVE's contents must
        be initialized with a call to subroutine COSTMI before the
        first call to routine COSTMF or COSTMB for a given transform
        length N.  WSAVE's contents may be re-used for subsequent
        calls to COSTMF and COSTMB with the same N.


LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG (REAL(N))) +4.


WORK    Real work array of dimension at least LENWRK.

LENWRK  Integer dimension of WORK array.  LENWRK must be at
        least LOT*(N+1).
```

Output Arguments

```
R       Real output array R.  For purposes of exposition,
        assume R's range of indices is given by
        R(0:(LOT-1)*JUMP+(N-1)*INC).

        The output values of R are written over the input values.
        For I=0,...,LOT-1 and J=0,...,N-1

         R(I*JUMP+J*INC) =

             N-1
```

```
               SUM  R(I*JUMP+N1*INC)*COS(J*N1*PI/(N-1))
               N1=0

IER       Integer error return
          =  0 successful exit
          =  1 input parameter LENR   not big enough
          =  2 input parameter LENSAV not big enough
          =  3 input parameter LENWRK not big enough
          =  4 input parameters INC,JUMP,N,LOT are not consistent.
          = 20 input error returned by lower level routine

               The parameters integers INC, JUMP, N and LOT are
               consistent if equality
               I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
               and J1,J2 < LOT implies I1=I2 and J1=J2.

               For multiple FFTs to execute correctly, input variables
               INC, JUMP, N and LOT must be consistent, otherwise at
               least one array element mistakenly is transformed more
               than once.
```

---

costmf

Return to Main Contents

NAME

COSTMF - real, multiple forward cosine fast Fourier transform

SYNOPSIS

```
 SUBROUTINE COSTMF (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine COSTMF computes the one-dimensional Fourier
 transform of multiple even sequences within a real array.  This
 transform is referred to as the forward transform or Fourier
 analysis, transforming the sequences from physical to spectral
 space.

 This transform is normalized since a call to COSTMF followed
 by a call to COSTMB (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences
         to be transformed.

 N       Integer length of each sequence to be transformed.  The
```

transform is most efficient when N-1 is a product of small primes.

INC Integer increment between the locations, in array R, of two consecutive elements within the same sequence.

R Real array containing LOT sequences, each having length N. R can have any number of dimensions, but the total number of locations must be at least LENR.

LENR Integer dimension of R array. LENR must be at least (LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must be initialized with a call to subroutine COSTMI before the first call to routine COSTMF or COSTMB for a given transform length N. WSAVE's contents may be re-used for subsequent calls to COSTMF and COSTMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least 2*N + INT(LOG (REAL(N))) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least LOT*(N+1).

Output Arguments

R Real output array R. For purposes of exposition, assume R's range of indices is given by R(0:(LOT-1)*JUMP+(N-1)*INC).

The output values of R are written over the input values.

```
For I=0,...,LOT-1
 R(I*JUMP) =

     0.5*X(I*JUMP)/(N-1)
```

```
           N-2
         + SUM  R(I*JUMP+*N1*INC)/(N-1)
           N1=1

         + 0.5*X(I*JUMP+(N-1)*INC)/(N-1)

      For I=0,...,LOT-1 and J=1,...,N-2
       R(I*JUMP+J*INC) =

           R(I*JUMP)/(N-1)

           N-2
         + SUM  2.0*(X(I*JUMP+*N1*INC)*COS(J*N1*PI/(N-1)))/(N-1)
           N1=1

         + ((-1)**J)*X(I*JUMP+(N-1)*INC)/(N-1)

      For I=0,...,LOT-1
       R(I*JUMP+(N-1)*INC) =

           0.5*X(I*JUMP)/(N-1)

           N-2
         + SUM  R(I*JUMP+*N1*INC)*((-1)**N1)/(N-1)
           N1=1

         + 0.5*((-1)**(N-1))*X(I*JUMP+(N-1)*INC)/(N-1)

IER   Integer error return
      =  0 successful exit
      =  1 input parameter LENR   not big enough
      =  2 input parameter LENSAV not big enough
      =  3 input parameter LENWRK not big enough
      =  4 input parameters INC,JUMP,N,LOT are not consistent.
      = 20 input error returned by lower level routine

           The parameters integers INC, JUMP, N and LOT are
           consistent if equality
           I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
           and J1,J2 < LOT implies I1=I2 and J1=J2.

           For multiple FFTs to execute correctly, input variables
```

INC, JUMP, N and LOT must be consistent, otherwise at least one array element mistakenly is transformed more than once.

sint1i

Return to Main Contents

NAME

SINT1I - initialization routine for SINT1B and SINT1F

SYNOPSIS

```
SUBROUTINE SINT1I (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine SINT1I initializes array WSAVE for use
in its companion routines SINT1F and SINT1B.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N+1 is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        N/2 + N + INT(LOG (REAL(N))) +4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines SINT1B or SINT1F.
```

```
IER      Integer error return
         =  0 successful exit
         =  2 input parameter LENSAV not big enough
         = 20 input error returned by lower level routine
```

---

sint1b

[Return to Main Contents]

NAME

SINT1B - real backward sine fast Fourier transform

SYNOPSIS

```
SUBROUTINE SINT1B (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine SINT1B computes the one-dimensional Fourier
transform of an odd sequence within a real array.  This transform
is referred to as the backward transform or Fourier synthesis,
transforming the sequence from spectral to physical space.

This transform is normalized since a call to SINT1B followed
by a call to SINT1F (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N+1 is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
        transformed.
```

```
LENR    Integer dimension of R array.  LENR must be at least
        INC*(N-1)+ 1.


WSAVE   Real work array of length LENSAV.  WSAVE's contents must
        be initialized with a call to subroutine SINT1I before the
        first call to routine SINT1F or SINT1B for a given transform
        length N.  WSAVE's contents may be re-used for subsequent
        calls to SINT1F and SINT1B with the same N.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        N/2 + N + INT(LOG (REAL(N))) +4.


WORK    Real work array of dimension at least LENWRK.

LENWRK  Integer dimension of WORK array.  Must be at least 2*N+2.

Output Arguments

 R      Real output array.  For purposes of exposition,
        assume R's range of indices is given by
        R(INC:N*INC).

        The output values of R are written over the input values.
        For J=1,...,N

         R(J*INC) =

              N
             SUM  R(N1*INC)*SIN(J*N1*PI/(N+1))
             N1=1

IER     Integer error return
        =  0 successful exit
        =  1 input parameter LENR   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        = 20 input error returned by lower level routine
```

sint1f

Return to Main Contents

NAME

SINT1F - real forward sine fast Fourier transform

SYNOPSIS

```
SUBROUTINE SINT1F (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine SINT1F computes the one-dimensional Fourier
transform of an odd sequence within a real array.  This transform
is referred to as the forward transform or Fourier analysis,
transforming the sequence from physical to spectral space.

This transform is normalized since a call to SINT1F followed
by a call to SINT1B (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N+1 is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
        transformed.
```

```
LENR    Integer dimension of R array.  LENR must be at least
        INC*(N-1)+ 1.


WSAVE   Real work array of length LENSAV.  WSAVE's contents must
        be initialized with a call to subroutine SINT1I before the
        first call to routine SINT1F or SINT1B for a given transform
        length N.  WSAVE's contents may be re-used for subsequent
        calls to SINT1F and SINT1B with the same N.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        N/2 + N + INT(LOG (REAL(N))) +4.


WORK    Real work array of dimension at least LENWRK.

LENWRK  Integer dimension of WORK array.  Must be at least 2*N+2.

Output Arguments

R       Real output array R.  For purposes of exposition,
        assume R's range of indices is given by R(INC:(N-1)*INC).

        The output values of R are written over the input values.
        For J=1,...,N

         R(J*INC) =

              N
             SUM  2.*R(N1*INC)*SIN(J*N1*PI/(N+1))/(N+1)
             N1=1

IER     Integer error return
        =  0 successful exit
        =  1 input parameter LENR   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        = 20 input error returned by lower level routine
```

sintmi

Return to Main Contents

NAME

SINTMI - initialization routine for SINTMB and SINTMF

SYNOPSIS

```
SUBROUTINE SINTMI (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine SINTMI initializes array WSAVE for use
in its companion routines SINTMF and SINTMB.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N       Integer length of each sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        N/2 + N + INT(LOG (REAL(N))) +4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines SINTMB or SINTMF.
```

```
IER       Integer error return
          =  0 successful exit
          =  2 input parameter LENSAV not big enough
          = 20 input error returned by lower level routine
```

sintmb

Return to Main Contents

NAME

SINTMB - real, multiple backward sine fast Fourier transform

SYNOPSIS

```
 SUBROUTINE SINTMB (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine SINTMB computes the one-dimensional Fourier
 transform of multiple odd sequences within a real array.  This
 transform is referred to as the backward transform or Fourier
 synthesis, transforming the sequences from spectral to physical
 space.

 This transform is normalized since a call to SINTMB followed
 by a call to SINTMF (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences.

 N       Integer length of each sequence to be transformed.  The
```

transform is most efficient when N+1 is a product of
small primes.

INC Integer increment between the locations, in array R, of two consecutive elements within the same sequence.

R Real array containing LOT sequences, each having length N. R can have any number of dimensions, but the total number of locations must be at least LENR.

LENR Integer dimension of R array. LENR must be at least (LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE Real work array of length LENSAV. WSAVE's contents must be initialized with a call to subroutine SINTMI before the first call to routine SINTMF or SINTMB for a given transform length N. WSAVE's contents may be re-used for subsequent calls to SINTMF and SINTMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least N/2 + N + INT(LOG (REAL(N))) +4.

WORK Real work array of dimension at least LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least LOT*(2*N+4).

Output Arguments

R Real output array. For purposes of exposition, assume R's range of indices is given by R(INC:(LOT-1)*JUMP+N*INC).

The output values of R are written over the input values. For I=0,...,LOT-1 and J=1,...,N

```
 R(I*JUMP+J*INC) =

     N
     SUM  R(I*JUMP+*N1*INC)*SIN(J*N1*PI/(N+1))
     N1=1
```

```
IER      Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big enough
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
         =  4 input parameters INC,JUMP,N,LOT are not consistent.
         = 20 input error returned by lower level routine

              The parameters integers INC, JUMP, N and LOT are
              consistent if equality
              I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
              and J1,J2 < LOT implies I1=I2 and J1=J2.

              For multiple FFTs to execute correctly, input variables
              INC, JUMP, N and LOT must be consistent ... otherwise at
              least one array element mistakenly is transformed more
              than once.
```

---

sintmf

Return to Main Contents

NAME

SINTMF - real, multiple forward sine fast Fourier transform

SYNOPSIS

```
 SUBROUTINE SINTMF (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine SINTMF computes the one-dimensional Fourier
 transform of multiple odd sequences within a real array.
 This transform is referred to as the forward transform or Fourier
 analysis, transforming the sequences from physical to spectral
 space.

 This transform is normalized since a call to SINTMF followed
 by a call to SINTMB (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences
         to be transformed.
```

```
N       Integer length of each sequence to be transformed.  The
        transform is most efficient when N+1 is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the same sequence.

R       Real array containing LOT sequences, each having length N.
        R can have any number of dimensions, but the total number
        of locations must be at least LENR.

LENR    Integer dimension of R array.  LENR must be at least
        (LOT-1)*JUMP + INC*(N-1)+ 1.


WSAVE   Real work array of length LENSAV.  WSAVE's contents must
        be initialized with a call to subroutine SINTMI before the
        first call to routine SINTMF or SINTMB for a given transform
        length N.  WSAVE's contents may be re-used for subsequent
        calls to SINTMF and SINTMB with the same N.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        N/2 + N + INT(LOG (REAL(N))) +4.


WORK    Real work array of dimension at least LENWRK.

LENWRK  Integer dimension of WORK array.  LENWRK must be at
        least LOT*(2*N+4).

Output Arguments

R       Real output array R.  For purposes of exposition,
        assume R's range of indices is given by
        R(0:(LOT-1)*JUMP+(N-1)*INC).

        The output values of R are written over the input values.
        For I=0,...,LOT-1 and J=1,...,N

         R(I*JUMP+J*INC) =

             N
             SUM  2.*R(I*JUMP+*N1*INC)*SIN(J*N1*PI/(N+1))/(N+1)
```

```
             N1=1

IER      Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big enough
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
         =  4 input parameters INC,JUMP,N,LOT are not consistent.

              The parameters integers INC, JUMP, N and LOT are
              consistent if equality
              I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
              and J1,J2 < LOT implies I1=I2 and J1=J2.

              For multiple FFTs to execute correctly, input variables
              INC, JUMP, N and LOT must be consistent ... otherwise at
              least one array element mistakenly is transformed more
              than once.
```

cosq1i

Return to Main Contents

NAME

COSQ1I - initialization routine for COSQ1B and COSQ1F

SYNOPSIS

```
SUBROUTINE COSQ1I (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine COSQ1I initializes array WSAVE for use
in its companion routines COSQ1F and COSQ1B.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG (REAL(N))) +4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines COSQ1B or COSQ1F.
```

```
IER      Integer error return
         =  0 successful exit
         =  2 input parameter LENSAV not big enough
         = 20 input error returned by lower level routine
```

cosq1b

Return to Main Contents

NAME

COSQ1B - real, backward quarter-cosine fast Fourier transform

SYNOPSIS

```
SUBROUTINE COSQ1B (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine COSQ1B computes the one-dimensional Fourier
transform of a sequence which is a cosine series with odd wave
numbers.  This transform is referred to as the backward transform
or Fourier synthesis, transforming the sequence from spectral to
physical space.

This transform is normalized since a call to COSQ1B followed
by a call to COSQ1F (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer number of elements to be transformed in the
        sequence. The transform is most efficient when N is a
        product of small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
        transformed.
```

LENR   Integer dimension of R array. LENR must be at least
        INC*(N-1)+ 1.

WSAVE   Real work array of length LENSAV. WSAVE's contents must
        be initialized with a call to subroutine COSQ1I before the
        first call to routine COSQ1F or COSQ1B for a given transform
        length N. WSAVE's contents may be re-used for subsequent
        calls to COSQ1F and COSQ1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least
        2*N + INT(LOG (REAL(N))) +4.

WORK   Real array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at
        least N.

Output Arguments

R   Real output array. For purposes of exposition,
        assume R's range of indices is given by
        R(0:(N-1)*INC).

        The output values of R are written over the input values.
        For J=0,...,N-1

```
 R(J*INC) =

     N-1
     SUM  R(N1*INC)*COS(J*(2*N1+1)*PI/(2*N))
     N1=0
```

WSAVE   Contains values initialized by subroutine COSQ1I that
        must not be destroyed between calls to routine COSQ1F
        or COSQ1B.

IER   Integer error return

```
= 0 successful exit
= 1 input parameter LENR   not big enough
= 2 input parameter LENSAV not big enough
= 3 input parameter LENWRK not big enough
= 20 input error returned by lower level routine
```

cosq1f

Return to Main Contents

NAME

COSQ1F - real, forward quarter-cosine fast Fourier transform

SYNOPSIS

```
SUBROUTINE COSQ1F (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine COSQ1F computes the one-dimensional Fourier
transform of a sequence which is a cosine series with odd wave
numbers.  This transform is referred to as the forward transform
or Fourier analysis, transforming the sequence from physical to
spectral space.

This transform is normalized since a call to COSQ1F followed
by a call to COSQ1B (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
        transformed.
```

LENR   Integer dimension of R array. LENR must be at least INC*(N-1)+ 1.

WSAVE   Real work array with dimension LENSAV. WSAVE's contents must be initialized with a call to subroutine COSQ1I before the first call to routine COSQ1F or COSQ1B for a given transform length N. WSAVE's contents may be re-used for subsequent calls to COSQ1F and COSQ1B with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least 2*N + INT(LOG (REAL(N))) +4.

WORK   Real array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least N.

Output Arguments

R   Real output array R. For purposes of exposition, assume R's range of indices is given by R(0:(N-1)*INC).

The output values of R are written over the input values. For J=0,...,N-1

```
 R(J*INC) =

     R(0)/N

     N-1
   + SUM  2.*R(N1*INC)*COS((2*J+1)*N1*PI/(2*N))/N
     N1=1
```

WSAVE   Contains values initialized by subroutine COSQ1I that must not be destroyed between calls to routine COSQ1F or COSQ1B.

```
IER      Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big enough
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
         = 20 input error returned by lower level routine
```

cosqmi

Return to Main Contents

NAME

COSQMI - initialization routine for COSQMB and COSQMF

SYNOPSIS

```
SUBROUTINE COSQMI (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine COSQMI initializes array WSAVE for use
in its companion routines COSQMF and COSQMB.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N       Integer length of each sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG (REAL(N))) +4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines COSQMB or COSQMF.
```

```
IER      Integer error return
         =  0 successful exit
         =  2 input parameter LENSAV not big enough
         = 20 input error returned by lower level routine
```

cosqmb

Return to Main Contents

NAME

COSQMB - real, multiple backward quarter-cosine fast Fourier transform

SYNOPSIS

```
 SUBROUTINE COSQMB (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine COSQMB computes the one-dimensional Fourier
 transform of multiple sequences, each of which is a cosine series
 with odd wave numbers.  This transform is referred to as the
 backward transform or Fourier synthesis, transforming the sequences
 from spectral to physical space.

 This transform is normalized since a call to COSQMB followed
 by a call to COSQMF (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences
         to be transformed.

 N       Integer length of each sequence to be transformed.  The
```

transform is most efficient when N is a product of small primes.

INC Integer increment between the locations, in array R, of two consecutive elements within the same sequence.

R Real array containing LOT sequences, each having length N. R can have any number of dimensions, but the total number of locations must be at least LENR.

LENR Integer dimension of R array. LENR must be at least (LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE Real work array with dimension LENSAV. WSAVE's contents must be initialized with a call to subroutine COSQMI before the first call to routine COSQMF or COSQMB for a given transform length N. WSAVE's contents may be re-used for subsequent calls to COSQMF and COSQMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least 2*N + INT(LOG (REAL(N))) +4.

WORK Real array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least LOT*N.

Output Arguments

R Real output array. For purposes of exposition, assume R's range of indices is given by R(0:(LOT-1)*JUMP+(N-1)*INC).

The output values of R are written over the input values. For I=0,...,LOT-1 and J=0,...,N-1

R(I*JUMP+J*INC) =

N-1

```
         SUM  R(I*JUMP+N1*INC)*COS(J*(2*N1+1)*PI/(2*N))
         N1=0

WSAVE   Contains values initialized by subroutine COSQMI that
        must not be destroyed between calls to routine COSQMF
        or COSQMB.

IER     Integer error return
        =  0 successful exit
        =  1 input parameter LENR   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        =  4 input parameters INC,JUMP,N,LOT are not consistent.
        = 20 input error returned by lower level routine

             The parameters integers INC, JUMP, N and LOT are
             consistent if equality
             I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
             and J1,J2 < LOT implies I1=I2 and J1=J2.

             For multiple FFTs to execute correctly, input variables
             INC, JUMP, N and LOT must be consistent, otherwise at
             least one array element mistakenly is transformed more
             than once.
```

cosqmf

Return to Main Contents

NAME

COSQMF - real, multiple forward quarter-cosine fast Fourier transform

SYNOPSIS

```
 SUBROUTINE COSQMF (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine COSQMF computes the one-dimensional Fourier
 transform of multiple sequences within a real array, where each
 of the sequences is a cosine series with odd wave numbers.  This
 transform is referred to as the forward transform or Fourier
 synthesis, transforming the sequences from spectral to physical
 space.

 This transform is normalized since a call to COSQMF followed
 by a call to COSQMB (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences
         to be transformed.
```

N   Integer length of each sequence to be transformed. The transform is most efficient when N is a product of small primes.

INC   Integer increment between the locations, in array R, of two consecutive elements within the same sequence.

R   Real array containing LOT sequences, each having length N. R can have any number of dimensions, but the total number of locations must be at least LENR.

LENR   Integer dimension of R array. LENR must be at least (LOT-1)*JUMP + INC*(N-1)+ 1.

WSAVE   Real work array o length LENSAV. WSAVE's contents must be initialized with a call to subroutine COSQMI before the first call to routine COSQMF or COSQMB for a given transform length N. WSAVE's contents may be re-used for subsequent calls to COSQMF and COSQMB with the same N.

LENSAV Integer dimension of WSAVE array. LENSAV must be at least 2*N + INT(LOG (REAL(N))) +4.

WORK   Real array of dimension LENWRK.

LENWRK Integer dimension of WORK array. LENWRK must be at least LOT*N.

Output Arguments

R   Real output array R. For purposes of exposition, assume R's range of indices is given by R(0:(LOT-1)*JUMP+(N-1)*INC).

The output values of R are written over the input values. For I=0,...,LOT-1 and J=0,...,N-1

R(I*JUMP+J*INC) =

R(I*JUMP)/N

```
             N-1
           + SUM  2.*R(I*JUMP+*N1*INC)*COS((2*J+1)*N1*PI/(2*N))/N
             N1=1

IER      Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big enough
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
         =  4 input parameters INC,JUMP,N,LOT are not consistent.
         = 20 input error returned by lower level routine

             The parameters integers INC, JUMP, N and LOT are
             consistent if equality
             I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
             and J1,J2 < LOT implies I1=I2 and J1=J2.

             For multiple FFTs to execute correctly, input variables
             INC, JUMP, N and LOT must be consistent, otherwise at
             least one array element mistakenly is transformed more
             than once.
```

sinq1i

Return to Main Contents

NAME

SINQ1I - initialization routine for SINQ1B and SINQ1F

SYNOPSIS

```
SUBROUTINE SINQ1I (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine SINQ1I initializes array WSAVE for use
in its companion routines SINQ1F and SINQ1B.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG (REAL(N))) +4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines SINQ1B or SINQ1F.
```

```
IER      Integer error return
         =  0 successful exit
         =  2 input parameter LENSAV not big enough
         = 20 input error returned by lower level routine
```

sinq1b

Return to Main Contents

NAME

SINQ1B - real backward quarter-sine fast Fourier transform

SYNOPSIS

```
SUBROUTINE SINQ1B (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine SINQ1B computes the one-dimensional Fourier
transform of a sequence which is a sine series with odd wave
numbers.  This transform is referred to as the backward transform
or Fourier synthesis, transforming the sequence from spectral to
physical space.

This transform is normalized since a call to SINQ1B followed
by a call to SINQ1F (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
```

```
        transformed.

LENR    Integer dimension of R array.  LENR must be at least
        INC*(N-1)+ 1.


WSAVE   Real work array of length LENSAV.  WSAVE's contents must
        be initialized with a call to subroutine SINQ1I before the
        first call to routine SINQ1F or SINQ1B for a given transform
        length N.  WSAVE's contents may be re-used for subsequent
        calls to SINQ1F and SINQ1B with the same N.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG (REAL(N))) +4.


WORK    Real work array of dimension at least LENWRK.

LENWRK  Integer dimension of WORK array.  LENWRK must be at least N.


Output Arguments

 R      Real output array R.  For purposes of exposition,
        assume R's range of indices is given by
        R(INC:N*INC).

        The output values of R are written over the input values.
        For J=1,...,N

         R(J*INC) =

              N
             SUM  R(N1*INC)*SIN(J*(2*N1-1)*PI/(2*N))
             N1=1

IER     Integer error return
        =  0 successful exit
        =  1 input parameter LENR   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        = 20 input error returned by lower level routine
```

sinq1f

Return to Main Contents

NAME

SINQ1F - real forward quarter-sine fast Fourier transform

SYNOPSIS

```
SUBROUTINE SINQ1F (N, INC, R, LENR, WSAVE, LENSAV, WORK, LENWRK, IER)

INTEGER    N, INC, LENR, LENSAV, LENWRK, IER
REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
FFTPACK 5.0 routine SINQ1F computes the one-dimensional Fourier
transform of a sequence which is a sine series of odd wave numbers.
This transform is referred to as the forward transform or Fourier
analysis, transforming the sequence from physical to spectral space.

This transform is normalized since a call to SINQ1F followed
by a call to SINQ1B (or vice-versa) reproduces the original
array  within roundoff error.

Input Arguments

N       Integer length of the sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the sequence.

R       Real array of length LENR containing the sequence to be
        transformed.
```

```
LENR     Integer dimension of R array.  LENR must be at least
         INC*(N-1)+ 1.


WSAVE    Real work array of length LENSAV.  WSAVE's contents must
         be initialized with a call to subroutine SINQ1I before the
         first call to routine SINQ1F or SINQ1B for a given transform
         length N.  WSAVE's contents may be re-used for subsequent
         calls to SINQ1F and SINQ1B with the same N.

LENSAV   Integer dimension of WSAVE array.  LENSAV must be at least
         2*N + INT(LOG (REAL(N))) +4.


WORK     Real work array of dimension at least LENWRK.

LENWRK   Integer dimension of WORK array.  LENWRK must be at least N.


Output Arguments

R        Real output array R.  For purposes of exposition,
         assume R's range of indices is given by
         R(INC:N*INC).

         The output values of R are written over the input values.
         For J=1,...,N

          R(J*INC) =

              N-1
            + SUM  (2.*R(N1*INC)*SIN(((2*J-1)*N1*PI/(2*N)))/N
              N1=1

            + ((-1)**(J+1))*R(N*INC)/N

IER      Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big enough
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
         = 20 input error returned by lower level routine
```

sinqmi

[Return to Main Contents]

NAME

SINQMI - initialization routine for SINQMB and SINQMF

SYNOPSIS

```
SUBROUTINE SINQMI (N, WSAVE, LENSAV, IER)
INTEGER    N, LENSAV, IER
REAL       WSAVE(LENSAV)
```

DESCRIPTION

```
FFTPACK 5.0 subroutine SINQMI initializes array WSAVE for use
in its companion routines SINQMF and SINQMB.  The prime factor-
ization of N together with a tabulation of the trigonometric
functions are computed and stored in array WSAVE.  Separate
WSAVE arrays are required for different values of N.

Input Arguments

N       Integer length of each sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG (REAL(N))) +4.


Output Arguments

WSAVE   Real work array with dimension LENSAV, containing the
        prime factors of N and also containing certain trigonometric
        values which will be used in routines SINQMB or SINQMF.
```

```
IER     Integer error return
        =  0 successful exit
        =  2 input parameter LENSAV not big enough
        = 20 input error returned by lower level routine
```

sinqmb

Return to Main Contents

NAME

SINQMB - real, multiple backward quarter-sine fast Fourier transform

SYNOPSIS

```
 SUBROUTINE SINQMB (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine SINQMB computes the one-dimensional Fourier
 transform of multiple sequences within a real array, where each
 of the sequences is a sine series with odd wave numbers.  This
 transform is referred to as the backward transform or Fourier
 synthesis, transforming the sequences from spectral to physical
 space.

 This transform is normalized since a call to SINQMB followed
 by a call to SINQMF (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences
         to be transformed.
```

```
N        Integer length of each sequence to be transformed.  The
         transform is most efficient when N is a product of
         small primes.

INC      Integer increment between the locations, in array R,
         of two consecutive elements within the same sequence.

R        Real array containing LOT sequences, each having length N.
         R can have any number of dimensions, but the total number
         of locations must be at least LENR.

LENR     Integer dimension of R array.  LENR must be at least
         (LOT-1)*JUMP + INC*(N-1)+ 1.


WSAVE    Real work array of length LENSAV.  WSAVE's contents must
         be initialized with a call to subroutine SINQMI before the
         first call to routine SINQMF or SINQMB for a given transform
         length N.  WSAVE's contents may be re-used for subsequent
         calls to SINQMF and SINQMB with the same N.

LENSAV   Integer dimension of WSAVE array.  LENSAV must be at least
         2*N + INT(LOG (REAL(N))) +4.


WORK     Real work array of dimension at least LENWRK.

LENWRK   Integer dimension of WORK array.  LENWRK must be at
         least LOT*N.


Output Arguments

 R       Real output array R.  For purposes of exposition,
         assume R's range of indices is given by
         R(INC:(LOT-1)*JUMP+N*INC).

         The output values of R are written over the input values.
         For I=0,...,LOT-1 and J=1,...,N

          R(I*JUMP+J*INC) =
```

```
              N
              SUM  R(I*JUMP+N1*INC)*SIN(J*(2*N1-1)*PI/(2*N))
              N1=1

 IER     Integer error return
         =  0 successful exit
         =  1 input parameter LENR   not big enough
         =  2 input parameter LENSAV not big enough
         =  3 input parameter LENWRK not big enough
         =  4 input parameters INC,JUMP,N,LOT are not consistent.
         = 20 input error returned by lower level routine

              The parameters integers INC, JUMP, N and LOT are
              consistent if equality
              I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
              and J1,J2 < LOT implies I1=I2 and J1=J2.

              For multiple FFTs to execute correctly, input variables
              INC, JUMP, N and LOT must be consistent ... otherwise at
              least one array element mistakenly is transformed more
              than once.
```

sinqmf

Return to Main Contents

NAME

SINQMF - real, multiple forward quarter-sine fast Fourier transform

SYNOPSIS

```
 SUBROUTINE SINQMF (LOT, JUMP, N, INC, R, LENR, WSAVE, LENSAV,
1                   WORK, LENWRK, IER)

 INTEGER    LOT, JUMP, N, INC, LENR, LENSAV, LENWRK, IER
 REAL       R(LENR), WSAVE(LENSAV), WORK(LENWRK)
```

DESCRIPTION

```
 FFTPACK 5.0 routine SINQMF computes the one-dimensional Fourier
 transform of multiple sequences within a real array, where each
 sequence is a sine series with odd wave numbers.  This transform
 is referred to as the forward transform or Fourier synthesis,
 transforming the sequences from spectral to physical space.

 This transform is normalized since a call to SINQMF followed
 by a call to SINQMB (or vice-versa) reproduces the original
 array  within roundoff error.

 Input Arguments

 LOT     Integer number of sequences to be transformed within
         array R.

 JUMP    Integer increment between the locations, in array R,
         of the first elements of two consecutive sequences
         to be transformed.
```

```
N       Integer length of each sequence to be transformed.  The
        transform is most efficient when N is a product of
        small primes.

INC     Integer increment between the locations, in array R,
        of two consecutive elements within the same sequence.

R       Real array containing LOT sequences, each having length N.
        R can have any number of dimensions, but the total number
        of locations must be at least LENR.

LENR    Integer dimension of R array.  LENR must be at least
        (LOT-1)*JUMP + INC*(N-1)+ 1.


WSAVE   Real work array of length LENSAV.  WSAVE's contents must
        be initialized with a call to subroutine SINQMI before the
        first call to routine SINQMF or SINQMB for a given transform
        length N.  WSAVE's contents may be re-used for subsequent
        calls to SINQMF and SINQMB with the same N.

LENSAV  Integer dimension of WSAVE array.  LENSAV must be at least
        2*N + INT(LOG (REAL(N))) +4.


WORK    Real work array of dimension at least LENWRK.

LENWRK  Integer dimension of WORK array.  LENWRK must be at
        least LOT*N.


Output Arguments

R       Real output array R.  For purposes of exposition,
        assume R's range of indices is given by
        R(INC:(LOT-1)*JUMP+N*INC).

        The output values of R are written over the input values.
        For I=0,...,LOT-1 and J=1,...,N

         R(I*JUMP+J*INC) =

             N-1
```

```
          + SUM  (2.*R(I*JUMP+*N1*INC)*SIN(((2*J-1)*N1*PI/(2*N)))/N
            N1=1

          + ((-1)**(J+1))*R(I*JUMP+N*INC)/N

IER     Integer error return
        =  0 successful exit
        =  1 input parameter LENR   not big enough
        =  2 input parameter LENSAV not big enough
        =  3 input parameter LENWRK not big enough
        =  4 input parameters INC,JUMP,N,LOT are not consistent.
        = 20 input error returned by lower level routine

             The parameters integers INC, JUMP, N and LOT are
             consistent if equality
             I1*INC + J1*JUMP = I2*INC + J2*JUMP for I1,I2 < N
             and J1,J2 < LOT implies I1=I2 and J1=J2.

             For multiple FFTs to execute correctly, input variables
             INC, JUMP, N and LOT must be consistent ... otherwise at
             least one array element mistakenly is transformed more
             than once.
```